

## 1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1 ,Class XI)
Module Name/Title	Unit 3, Module 7, Solving problems using Newton's laws of motion Chapter 5, Laws of motion
Module Id	Keph_10507_eContent
Pre-requisites	Kinematics, motion in one and two dimension equations of motion Laws of motion, vector algebra, basic trigonometry,
Objectives	<p>After going through this module you will be able to</p> <ul style="list-style-type: none"> <li>• Apply laws of motion to motion in one and two dimensions</li> <li>• Correlate kinematics and dynamics</li> <li>• Understand forces we encounter in real life</li> <li>• Visualize concurrent forces and equilibrium of bodies under multiple forces</li> <li>• Draw free body diagrams for forces</li> <li>• Solve problems</li> </ul>
Keywords	External force, Inertia, Net Force ,free body diagram, equilibrium of forces, normal force, tension, weight, friction

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## 1. UNIT SYLLABUS

### Chapter 5: Laws of Motion

Intuitive concept of force, Inertia, Newton's first law of motion, momentum and Newton's second law of motion, Impulse; Newton's third law of motion.

Law of conservation of linear momentum and its applications.

Equilibrium of concurrent forces, Static and kinetic friction, laws of friction, rolling friction, lubrication.

Dynamics of uniform circular motion: Centripetal force, examples of circular motion (vehicle on a level circular road, vehicle on banked road).

**2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS****7 Modules**

The above unit is divided into seven modules as follows:

Module 1	<ul style="list-style-type: none"> <li>• Force</li> <li>• inertia</li> <li>• First law of motion</li> </ul>
Module 2	<ul style="list-style-type: none"> <li>• Momentum</li> <li>• Second law</li> <li>• Impulse</li> <li>• <math>F=ma</math></li> <li>• Constant and variable force</li> </ul>
Module 3	<ul style="list-style-type: none"> <li>• Third law</li> <li>• Conservation of linear momentum and its applications</li> </ul>
Module 4	<ul style="list-style-type: none"> <li>• Types of forces (tension, normal, weight, ...)</li> <li>• Equilibrium of concurrent forces</li> <li>• Free</li> </ul>
Module 5	<ul style="list-style-type: none"> <li>• Friction</li> <li>• Coefficient of friction</li> <li>• Static friction</li> <li>• Kinetic friction</li> <li>• Rolling friction</li> <li>• Role of friction in daily life</li> </ul>
Module 6	<ul style="list-style-type: none"> <li>• Dynamics of circular motion</li> <li>• Centripetal force</li> <li>• Banking of roads</li> </ul>
Module 7	Using laws of motion to solve problems in daily life

**MODULE 7**

### 3. WORDS YOU MUST KNOW

- **Rest:** A body is said to be at rest if it does not change its position with time with respect to its surroundings.
- **Motion:** A body is said to be in motion if it changes its position with time with respect to its surroundings.
- **Point object:** If the position of an object changes by distances much larger than the dimensions of the body the body may be treated as a point object.
- **Frame of reference:** Any reference frame, the coordinates (x, y, z) of which indicate the change in position of object with time.
- **Inertial frame:** Is a stationary frame of reference or one moving with constant speed observer someone who is observing objects.
- **Distance travelled:** The distance an object has moved from its starting position. Its SI unit is m and it can be zero or positive.
- **Displacement:** The distance an object has moved from its starting position moves in a particular direction. Its SI unit is m and it can be zero, positive or negative.
- **Path length:** Actual distance travelled is called the path length.
- **Position- time, distance -time, displacement- time graph:** These graphs are used for showing at a glance the position, distance travelled or displacement versus time elapsed.
- **Speed:** Rate of change of position is called speed. Its SI unit is m/s.
- **Time elapsed:** It is time interval between any two observations of an object whether stationary or moving.
- **Velocity:** The time rate of change of displacement is called velocity.

- **Uniform motion:** When a particle has equal displacements, in equal intervals of time, (howsoever small this time interval may be) it is said to have a uniform motion. The acceleration for a particle in uniform motion would be zero.
- **Momentum (p):** An indicator of the impact capacity of a moving body. We have  $\mathbf{p} = m\mathbf{v}$ . **It is a vector quantity, its unit is kg m/s.**
- **Acceleration:** Time rate of change of velocity of a particle, equals its acceleration. It can be positive negative or zero. SI unit is  $\text{ms}^{-2}$ .
- **Vector:** A physical quantity that needs both a magnitude and a direction for its specification.
- **Vector Algebra:** The branch of mathematics that deals with computations involving addition, subtraction, and multiplication of vectors.
- **Force:** A body will continue in its state of rest, or uniform motion until and unless it is acted upon by an external unbalanced force.
- **Inertia:** An inherent property of all objects; an object continues in its state of rest or uniform motion unless and until a non-zero external force acts on it.
- **Impulse:** change of momentum due to a force acting for a short duration of time  

$$Ft = mv - mu$$
- **Static friction:** The force of friction which comes into play between the surfaces of two bodies before the body actually starts moving is called static friction.
- **Kinetic friction:** The force of friction acting between the two surfaces, when one surface is in steady motion over the other surface is called kinetic friction.
- **Centripetal force:** A force on a body moving in circle acting towards the centre of the circle.
- **Angular velocity:** A vector quantity describing the speed in terms of angular displacement of an object in circular motion and the direction is perpendicular to the plane of its circular motion. It is given by:  $v = \omega r$

- **Rigid body:** An object for which individual particles continue to be at the same separation over a period of time.
- **Motion in one dimension:** When the position of an object can be shown by change in any one coordinate out of the three coordinates (x, y, z), also called motion in a straight line.
- **Motion in two dimensions:** When the position of an object can be shown by changes any two coordinate out of the three coordinates (x, y, z), also called motion in a plane.
- **Motion in three dimension:** When the position of an object can be shown by changes in all three coordinate out of the three (x, y, z).
- **Average speed:** It is the ratio of the total path length to the total time taken for the change in position. **three laws of motion**
- **Velocity:** Rate of change of position in a particular direction is called velocity. It can be zero, negative and positive and its SI unit is m/s.
- **Velocity time graph:** It is a graph showing change of velocity with time. This graph can be obtained from position- time graphs.
- **Acceleration:** Time rate of change of velocity.
- **Acceleration time graph:** A graph showing change velocity with time. This graph can be obtained from position- time, velocity- time graphs.

- **Equations of motion:** Equations of motion are the mathematical relation between initial velocity (u), final velocity (v), acceleration(a), distance travelled(s) and time elapsed (t). These are:

$$(a) v = u + at$$

$$(b) v^2 = u^2 + 2as$$

$$(c) s = ut + \frac{1}{2}at^2$$

- **Newton's laws of motion:** The three laws of motion are:

(a) An object will continue to be in its state of rest or uniform motion along a straight line, unless and until an unbalanced force is applied on it. Therefore, every object continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise. If the net external force on a body is zero, its acceleration is zero. Acceleration can be non- zero only if there is a net external force on the body.

(b) The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts the same force for the same time causes the same change in momentum for different bodies. Any internal forces in the system are not to be included in F.

$$F = m a = \frac{m(v-u)}{t}$$

$$\text{Or } F t = mv - mu$$

(c) To every action, there is always an equal and opposite reaction. Forces always occur in pairs. Force on a body A by body B is equal and opposite to the force on the body B by A.

- **Projectile motion:** If a particle is projected and it moves under the influence of gravity, its motion is parabolic. It has a horizontal maximum range depending upon initial

velocity, angle of projection and place/location on earth where it is projected (value of  $g$  changes).

- **Circular motion:** When the path of travel of a body is a circle.
- **Centripetal acceleration:** The body moving in a circular path has an acceleration towards the center of the circle, even if the speed of revolution is constant as

$$a = \frac{v^2}{r}$$

- **Centripetal force:** The force responsible for centripetal acceleration

$$F = m \frac{v^2}{r}$$

- **Law of conservation of linear momentum:** The total linear momentum of an isolated system of interacting particles is conserved.

#### 4. INTRODUCTION

In the earlier modules, you study how objects move or kinematics, you also learnt what causes a body to move and change its direction, speed or both in effect the dynamics of moving objects. In this unit on motion of rigid bodies we have studied ways to predict motion using equations of motion. You also understood the cause of motion. This motion could be along a straight line, in a plane or in a circle. What we also considered was causes of motion of body. The laws of motion relate force and acceleration. We understood the need for an unbalanced external force to cause a change in the condition of rest or uniform motion of a body along a straight line.

#### 5. COMMON FORCES IN MECHANICS

**THE GRAVITATIONAL FORCE** is, of course, all pervasive. Every object on the earth experiences the force of gravity due to the earth. Gravity also governs the motion of celestial bodies. The gravitational force can act at a distance without the need of any intervening medium.



- Weight of a body is the force due to earth on anybody keeping the second law of motion in mind  $F = ma$

here the force = weight or  $W = m$  (acceleration due to gravity)

or  $W = mg$

- This force acts at a distance
- For any mass it will change if the acceleration due to gravity changes.

You have studied in your earlier classes that the weight of a body on moon is only  $1/6$  of its weight on the surface of the earth. The variation will become clearer once you study the unit on gravitation.

**CONTACT FORCES** arise also when solids are in contact with fluids. For example, for a solid immersed in a fluid, there is an upward buoyant force equal to the weight of the fluid displaced. The viscous force or viscous drag, air resistance, etc are also examples of contact forces. Two other common forces are tension in a string and the force due to spring. When a spring is compressed or extended by an external force, a restoring force is generated. This force is usually proportional to the compression or elongation (for small displacements).

The **SPRING FORCE F** is written as  $F = -kx$  where  $x$  is the displacement and  $k$  is the force constant. The negative sign denotes that the force is opposite to the displacement from the up stretched state. For an inextensible string, the force constant is very high. The restoring force in a string is called tension. It is customary to use a constant tension  $T$  throughout the string. This assumption is true for a string of negligible mass and uniform area of cross-section.

We have learnt that there are four fundamental forces in nature. Of these, the weak and strong nuclear forces appear in domains that do not concern us here.

Only the gravitational and electrical forces (electromagnetic forces) are relevant in the context of mechanics.

The different contact forces of mechanics mentioned above fundamentally arise from electrical forces. This may seem surprising since we are talking of uncharged and non-

magnetic bodies in mechanics. At the microscopic level, all bodies are made of charged constituents (nuclei and electrons) and the various contact forces arising due to elasticity of bodies, molecular collisions and impacts, etc. can ultimately be traced to the electrical forces (electromagnetic forces) between the charged constituents of different bodies.

The detailed microscopic origin of these forces is, however, complex and not useful for handling problems in mechanics at the macroscopic scale.

This is why they are treated as different types of forces with their characteristic properties determined empirically.

### **FORCE OF FRICTION**

Let us return to the example of a body of mass  $m$  at rest on a horizontal table. The force of gravity ( $mg$ ) is cancelled by the normal force ( $N$ ) of the table. Now suppose a force  $F$  is applied horizontally to the body. We know from experience that a small applied force may not be enough to move the body. But if the applied force  $F$  were the only external force on the body, it will tend to move with acceleration  $F/m$ , however, small. Clearly, the body remains at rest because some other force comes into play in the horizontal direction and opposes the applied force  $F$ , resulting in zero net force on the body. This force  $f_s$  parallel to the surface of the body in contact with the table is known as frictional force, or simply friction. The subscript stands for static friction to distinguish it from kinetic friction  $f_k$  that we consider later. Note that static frictional force,  $f_s$  does not exist by itself. When there is no applied force, there is no static friction. It comes into play the moment there is an applied force. As the applied force  $F$  increases,  $f_s$  also increases, remaining equal and opposite to the applied force (up to a certain limit), keeping the body at rest. Hence, it is called static friction. Static friction opposes impending motion. The term impending motion means motion that would take place (but does not actually take place) under the applied force, if friction were absent. We know from experience that as the applied force exceeds a certain limit, the body begins to move. It is found experimentally that the limiting value of static friction  $(f_s)_{\max}$  is independent of the area of contact and varies with the normal force ( $N$ ) approximately as  $(f_s)_{\max} = \mu_s N$ , where  $\mu_s$  is a constant of proportionality depending only on the nature of the surfaces in contact. The constant  $\mu_s$  is called the coefficient of static friction.

The law of static friction may, thus, be written as  $f_s \leq \mu_s N$ . If the applied force  $F$  exceeds the  $(f_s)_{\max}$  body begins to slide on the surface. It is found experimentally that when relative motion has started, the frictional force decreases from the static maximum value. Frictional force that opposes relative motion between surfaces in contact is called kinetic or sliding friction and is denoted by  $f_k$ . Kinetic friction, like static friction, is found to be independent of the area of contact. Further, it is nearly independent of the velocity. It satisfies a law similar to that for static friction:  $f_k = \mu_k N$  where,  $\mu_k$  the coefficient of kinetic friction, depends only on the surfaces in contact. As mentioned above, experiments show that  $\mu_k$  is less than  $\mu_s$ . When relative motion has begun, the acceleration of the body according to the second law is  $\frac{(F-f_k)}{m}$ . For a body moving with constant velocity,  $a=0$ ,  $F = f_k$ . If the applied force on the body is removed, its acceleration is  $-\frac{f_k}{m}$  and it eventually comes to a stop. The laws of friction given above do not have the status of fundamental laws like those for gravitational, electric and magnetic forces. They are empirical relations that are only approximately true. Yet they are very useful in practical calculations in mechanics.

So, force may manifest itself as:

1. **Push or a pull**
2. **Weight**
3. **Tension in mass less (negligible mass) and inextensible (of negligible extension) strings**
4. **Normal force**
5. **Force of friction**

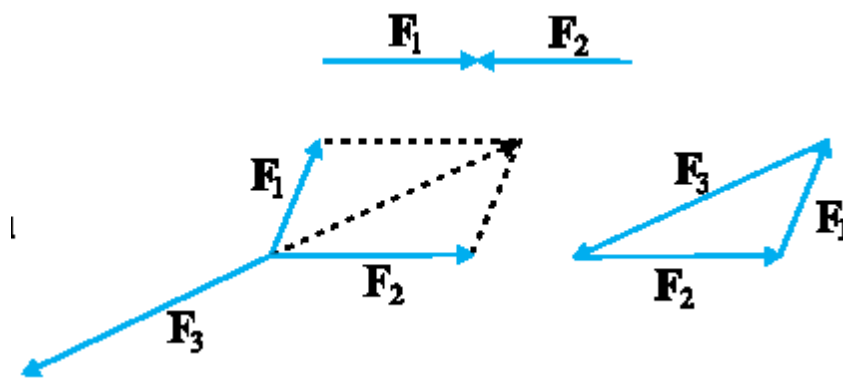
## **6. CONCURRENT FORCES**

Forces acting at the same point on a body are called concurrent forces.

Equilibrium of a particle, in mechanics, refers to the situation when the net external force on the particle is zero. According to the first law of motion, this means that, the particle is either at rest or in uniform motion along a straight line.

If two forces  $F_1$  and  $F_2$ , act on a particle, translational equilibrium requires  $F_1 = -F_2$  i.e. the two forces on the particle must be equal and opposite. Equilibrium under three concurrent forces  $F_1$ ,  $F_2$  and  $F_3$  requires that the vector sum of the three forces is zero.  $F_1 + F_2 + F_3 = 0$

In other words, the resultant of any two forces say  $F_1$  and  $F_2$ , obtained by the parallelogram law of forces must be equal and opposite to the third force,  $F_3$ .



The three forces in translational equilibrium can be represented by the sides of a triangle with the vector arrows taken in the same sense or order. The result can be generalised to any number of forces. A particle is in equilibrium under the action of forces  $F_1, F_2, \dots, F_n$  if they can be represented by the sides of a closed n-sided polygon with arrows directed in the same sense.

In three dimensions, we could put the equations as equation along x direction:

$$F_{1x} + F_{2x} + F_{3x} = 0$$

Along y-direction:

$$F_{1y} + F_{2y} + F_{3y} = 0$$

Along z-direction:

$$F_{1z} + F_{2z} + F_{3z} = 0$$

Where,  $F_{1x}$ ,  $F_{1y}$  and  $F_{1z}$  are the components of force  $F_1$  along x, y and z directions respectively.

## 7. TRANSLATIONAL EQUILIBRIUM

If the resultant of all concurrent forces is zero, the body is in translational equilibrium.

This means the body is either at rest or moves with constant velocity. Notice we are not saying constant speed but constant velocity.

## 8. CONCEPTUAL PROBLEMS BASED ON NEWTON'S LAWS OF MOTION

The **three laws of motion** that you have learnt in this chapter are the foundation of mechanics. You should now be able to handle a large variety of problems in mechanics. A typical problem in mechanics usually does not merely involve a single body under the action of given forces. More often, we will need to consider an assembly of different bodies exerting forces on each other. Besides, each body in the assembly experiences the force of gravity. When trying to solve a problem of this type, it is useful to remember the fact that we can choose any part of the assembly and apply the laws of motion to that part provided we include all forces on the chosen part due to the remaining parts of the assembly.

We may call the chosen part of the assembly as the system and the remaining part of the assembly (plus any other agencies of forces) as the environment.

### TO HANDLE A TYPICAL PROBLEM IN MECHANICS SYSTEMATICALLY, YOU SHOULD USE THE FOLLOWING STEPS:

- a. Draw a diagram showing schematically the various parts of the assembly of bodies, the links, supports, etc.
- b. Choose a convenient part of the assembly as one system.
- c. Draw a separate diagram which shows this system and all the forces on the system by the remaining part of the assembly. Include also the forces on the system by other agencies. Do not include the forces on the environment by the system. A diagram of this type is known as 'a free-body diagram'. (Note this does not imply that the system under consideration is without a net force). Free body diagram
- d. A diagram which shows all concurrent forces acting on the body at a point in order to make calculations easy
- e. In a free-body diagram, include information about forces (their magnitudes and directions) that are either given or you are sure of (e.g., the direction of tension in a string

along its length). The rest should be treated as unknowns to be determined using laws of motion.

- f. If necessary, follow the same procedure for another choice of the system. In doing so, employ Newton's third law of motion. That is, if in the free-body diagram of body A, the force on body A due to body B is shown as  $F$ , then in the free-body diagram of body B, the force on body B due to body A should be shown as  $-F$ .

The following example illustrates the above points more clearly.

(For simplicity in numerical calculations, take  $g = 10 \text{ m s}^{-2}$ )

**EXAMPLE:**

**Give the magnitude and direction of the net force acting on**

- (a) a drop of rain falling down with a constant speed,**
- (b) a cork of mass 10 g floating on water,**
- (c) a kite skilfully held stationary in the sky,**
- (d) a car moving with a constant velocity of 30 km/h on a rough road,**
- (e) a high-speed electron in space far from all material objects, and free of electric and magnetic fields.**

**SOLUTION**

In a, b, c, d and e there is no net force as the body is either stationary or moving with constant speed.

**EXAMPLE:**

**A pebble of mass 0.05 kg is thrown vertically upwards. Give the direction and magnitude of the net force on the pebble,**

- (a) during its upward motion,**
- (b) during its downward motion,**

(c) at the highest point where it is momentarily at rest.

(ignore air resistance) Do your answers change if the pebble was thrown at an angle of  $45^\circ$  with the horizontal direction?

### SOLUTION

The only force in each case is the force of gravity. Force acts vertically downwards only. The answers do not change even if we consider the pebble moving at an inclination.

### EXAMPLE:

Given the magnitude and direction of the net force acting on a stone of mass 0.1 kg.

(a) Just after it is dropped from the window of a stationary train,

(b) Just after it is dropped from the window of a train running at a constant velocity of 36 km/h,

(c) Just after it is dropped from the window of a train accelerating with  $1 \text{ m s}^{-2}$

### SOLUTION:

$$(F = m a = m g)$$

force of 1N vertically downwards in every case, as the force at an instant depends on the situation falling under gravity,  $a = g = 10 \text{ m/s}^2$  and not on history

### EXAMPLE:

A car of mass 1500 kg and a truck of mass 3000 kg are moving with a speed of 10 m/s which is equivalent to 36 km/h. The drivers see a herd of cows on the road ahead. They need to stop their vehicles within a distance of 50 m so as to save the animals'. How much force should the brakes apply in order to avoid any accident?

### SOLUTION:

$$\text{Force} = m a$$

So we need acceleration, 'a'.

Using an equation of motion to calculate 'a' we have,

$$V^2 = u^2 + 2as$$

Since the mass is known the force can then be calculated:

$$u = 10 \text{ m/s}$$

$$v = 0$$

$$s = 50 \text{ m}$$

$$0 = 100 + 2 \times 50 \times a$$

$$a = 1 \text{ m/s}^2$$

Mass of the truck = 3000 kg

$$F = 3000 \text{ N}$$

Mass of the car = 1500 kg

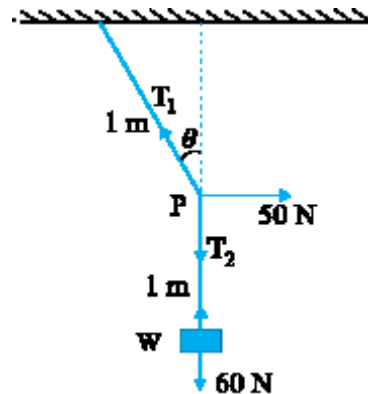
$$F = 1500 \text{ N.}$$

So, the truck needs a larger force to stop as compared to the car.

This problem will help you to understand why we have different speed limits for light and heavy vehicles.

**EXAMPLE:**

A bag of potatoes of mass 6 kg is suspended by a rope from the ceiling. A force of 50 N in the horizontal direction is applied at the midpoint of the rope. Find the angle that the rope must make to keep the potatoes in equilibrium.



**SOLUTION**



The FBD of the given situation is as follows:

Let us recognize the three forces

- (i) Weight
- (ii) the pull
- (iii) Tension

Three forces are acting simultaneously on the bag

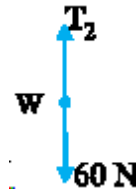
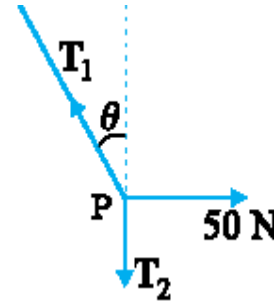
Let the rope make an angle  $\theta$  with the vertical

Resolving the forces horizontally and vertically we have

$$T \sin\theta = 50 \text{ N}$$

$$T \cos\theta = 6 \times 10 = 60 \text{ N}$$

$$\tan\theta = \frac{5}{6} \text{ or } \theta = \tan^{-1} \frac{5}{6} = 39.8^\circ$$



**EXAMPLE:**

A train is moving along a horizontal track. A pendulum suspended from the roof of the compartment makes an angle of  $4^\circ$  with the vertical. Obtain the acceleration of the train. (Take  $g = 10 \text{ m/s}^2$ )

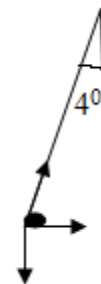
**SOLUTION:**

Two forces acting on the pendulum are:

- (i) Weight
- (ii) Tension in the string

Resolving T along the horizontal and vertical directions we find

$$T \cos 4^\circ = m g$$



Let the train have an acceleration of  $a$

$$T \sin 4^\circ = m a$$

$$\tan 4^\circ = \frac{a}{g}$$

$$a = g \tan 4^\circ = 10 \times 0.07 = 0.7 \text{ ms}^{-2}$$

**EXAMPLE:**

**Motion of connected bodies example** Connected systems are often seen as in trains, trailers on the road, Cranes at construction work sites etc.

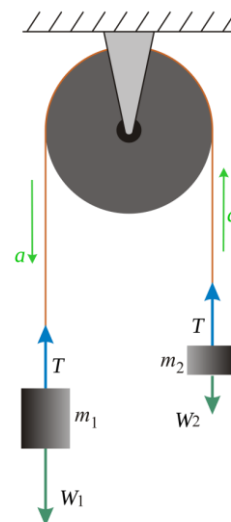
The system shown has two masses,  $m_1 = 10 \text{ kg}$  so

$$W_1 = m_1 g_m = 10 g \text{ and mass } m_2 = 12 \text{ kg}$$

$$\text{So, } W_2 = m_2 g_m = 12 g_m \text{ at its ends with } g_m = 10 \text{ ms}^{-2}$$

Calculate:

- a) Tension in the string
- b) Acceleration with which the system moves once 12 kg weight is released



**SOLUTION:**

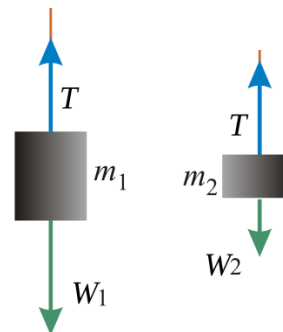
FBD of the given situation is as follows:

As  $W_2$  is greater than  $W_1$

Let the system move with an acceleration of

$a$  and the tension in the string is  $T$ .

$$W_2 - T = 12 a$$



$$T - W_1 = 10 a$$

From the above equations, we calculate the values of T and a as

$$a = 0.90 \text{ m s}^{-2}$$

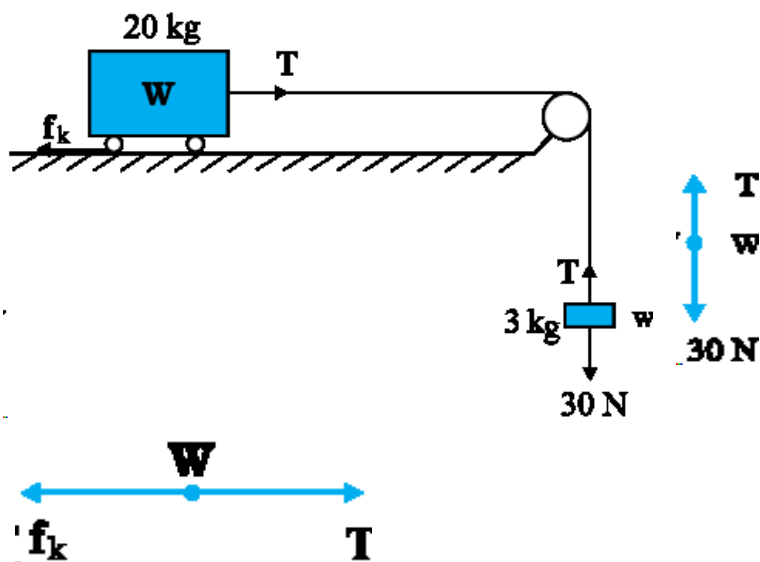
$$T = 109.09 \text{ N}$$

**EXAMPLE:**

What is the acceleration of the block and trolley system shown in the fig, if the coefficient of kinetic friction between the trolley and the surface is 0.04?

What is the tension in the string?

(Take  $g = 10 \text{ m s}^{-2}$ ). Neglect the mass of the string.



**SOLUTION:**

As the string is inextensible (negligible extension), and the pulley is smooth, the 3 kg block and the 20 kg trolley both have same magnitude of acceleration.

Applying second law to motion of the block

$$30 - T = 3a$$

Apply the second law to motion of the trolley

$$T - f_k = 20 a.$$

$$\text{Now } f_k = \mu_k N,$$

$$\text{Here } \mu_k = 0.04,$$

$$N = 20 \times 10$$

$$= 200 \text{ N.}$$

Thus the equation for the motion of the trolley is

$$T - 0.04 \times 200 = 20 a$$

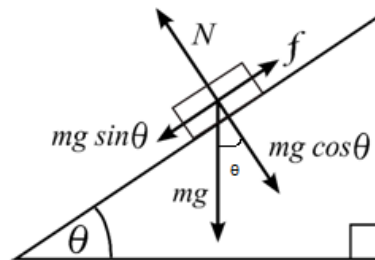
$$\text{Or } T - 8 = 20a.$$

These equations give  $a = \frac{22}{23} \text{ m s}^{-2} = 0.96 \text{ m s}^{-2}$  and  $T = 27.1 \text{ N}$ .

**EXAMPLE:**

Find the acceleration of the block of 100 g placed on the inclined plane, making an angle of  $30^\circ$ , with the horizontal as it slides down if the plane is:

- a. Friction less (negligible friction)
- b. If the coefficient of friction is  $\mu$



Forces on the block are shown below:

The forces acting on a block of mass  $m$  at rest on an inclined plane are

- (i) the weight  $mg$  acting vertically downwards
- (ii) the normal force  $N$  of the plane on the block, and
- (iii) the static frictional force  $f_s$  opposing the impending motion.

**SOLUTION**

In equilibrium, the resultant of these forces must be zero. Resolving the weight  $mg$  along the two directions shown, we have

$$m g \sin \theta = f_s, \quad m g \cos \theta = N$$

As  $\theta$  increases, the self-adjusting frictional force  $f_s$  increases until at  $\theta = \theta_{\max}$ ,  $f_s$  achieves its maximum value,  $f_{\max} = \mu_s N$ .

Therefore,

$$\tan \theta_{\max} = \mu_s \text{ or } \theta_{\max} = \tan^{-1} \mu_s$$

When  $\theta$  becomes just a little more than  $\theta_{\max}$ , there is a small net force on the block and it begins to slide.

Note that  $\theta_{\max}$  depends only on  $\mu_s$  and is independent of the mass of the block.

For  $\theta_{\max} = 15^\circ$ ,

$$\mu_s = \tan 15^\circ = 0.27$$

## 9. SUMMARY

**In this module, we have learnt:**

- **Force may manifest itself as:**
  - (a) Push or a pull
  - (b) Weight
  - (c) Tension in mass less (negligible mass) and inextensible (of negligible extension) strings
  - (d) Normal force
  - (e) Force of friction
- Forces acting at the same point on a body are called concurrent forces

- The resultant of all concurrent forces is zero the body is in translational equilibrium.
  
- **To handle a typical problem in mechanics systematically, you should use the following steps:**
  - a. Draw a diagram showing schematically the various parts of the assembly of bodies, the links, supports, etc.
  
  - b. Choose a convenient part of the assembly as one system.
  
  - c. Draw a separate diagram which shows this system and all the forces on the system by the remaining part of the assembly. Include also the forces on the system by other agencies. Do not include the forces on the environment by the system. A diagram of this type is known as ‘a free-body diagram’. (Note this does not imply that the system under consideration is without a net force).
  
  - d. Free body diagram: A diagram which shows all concurrent forces acting on the body at a point in order to make calculations easy
  
  - e. In a free-body diagram, include information about forces (their magnitudes and directions) that are either given or you are sure of (e.g., the direction of tension in a string along its length). The rest should be treated as unknowns to be determined using laws of motion.
  
  - f. If necessary, follow the same procedure for another choice of the system. In doing so, employ Newton’s third law of motion. That is, if in the free-body diagram of body A, the force on body A due to body B is shown as  $F$ , then in the free-body diagram of body B, the force on body B due to body A should be shown as  $-F$ .